Phantom energy accretion onto a black hole in Hořava Lifshitz gravity

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Abstract

In this Letter, we examine the phantom energy accretion onto a Kehagias-Sfetsos black hole in Hořava Lifshitz gravity. To discuss the accretion process onto the black hole, the equations of phantom flow near the black hole have been derived. It is found that mass of the black hole decreases because of phantom accretion. We discuss the conditions for critical accretion. Graphically, it has been found that the critical accretion phenomena is possible for different values of parameters. The results for the Schwarzschild black hole can be recovered in the limiting case.

Key Words: Phantom Energy, Accretion, Black Hole, Horava-Lifshitz Gravity.
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1 Introduction

Currently, observational cosmology has revealed that our universe is in an accelerating phase. It has been verified by the data of type-Ia Supernova and a large-scale structure [1-4]. Further, the anisotropic behavior of radiations in cosmic microwave background (CMB) as predicted by WMAP [5-7] favor the

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accelerating expansion of universe. The exotic energy with negative pressure, known as *dark energy* (DE), is responsible for accelerating behavior of the universe. Despite several observational facts, the nature of DE is still an open issue in physics.

It is well-known that when phantom energy from an external source accretes onto BH, then mass of BH decreases such that it eventually attains extremal state and finally converts to NS. During accretion process, charge and angular momentum remain unchanged. In Newtonian theory, the problem of accretion of matter onto the compact object was formulated by Bondi [8]. Michel [9] derived the relativistic formula for the accretion of perfect fluid onto the Schwarzschild BH. Babichev et al. [10] investigated that phantom accretion onto a BH that can decrease its the mass if the back reaction effects of accreting phantom fluid on geometry of BH are neglected. Jamil et al. [11] discussed the critical accretion on the RN BH. They determined the mass to charge ratio beyond which a BH can be converted into a NS. The same conclusion was drawn by Babichev et al. [12], by using the linear EoS and Chaplygin gas EoS for RN BH. Madrid and Gonzalez [13] showed that accreting phantom energy onto Kerr BH can convert it into a NS. Sharif and Abbas [14-16] discussed the phantom energy accretion onto a class of BHs and found that CCH is valid for phantom accretion onto a stringy charged BH.

Motivated by the recent development in Horava-Lifshitz gravity, we investigate the phantom accretion onto a static Kehagias-Sfetsos (KS) BH in Hořava Lifshitz (HL) gravity, by using Babichev-Dokuchaev-Eroshenko method [17] and discuss the locations of the critical points of accretion. Further, the relations between critical points and horizons have been found. The gravitational units are used. All the Latin and Greek indices vary from 0 to 3, otherwise it will be stated.

2 Phantom Energy Accretion Onto a BH in Horava-Lifshitz Gravity

The KS BH solution [17] is given by

$$ds^2 = A(r)dt^2 - \frac{1}{A(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

where $A(r)=1+wr^2-wr^2\sqrt{1+\frac{4m}{wr^3}}$ and $w=\frac{16\mu^2}{\kappa^2}$ (μ and κ are constants), for $w\rightarrow\infty$, KS BH $\rightarrow$ Schwarzschild BH. The horizons of KS BH can be obtained by solving $A(r)=0$, for r. Hence, in this case horizons are given by

$$r_{\pm}=m\left(1\pm\sqrt{1-\frac{1}{2wm^2}}\right). \tag{2}$$

The energy momentum tensor representing phantom energy in perfect fluid given by

$$T_{\mu\nu}=(\rho+p)u_\mu u_\nu-pg_{\mu\nu}, \tag{3}$$

where ρ and p are energy density and pressure of phantom energy and $u^\mu=(u^t,u^r,0,0)$ is the velocity four-vector of fluid flow. We note that u^μ satisfies the normalization condition, that is $u^\mu u_\mu=1$.

For the phantom energy accretion onto a KS BH, we shall drive two equations of motions, one by the conservation of energy-momentum tensor and other by projecting the energy-momentum conservation law on the four-velocity. The energy conservation equation $T^{0\mu}_{;\mu}=0$ is given by

$$r^2u(\rho+p)\left(1+wr^2-wr^2\sqrt{1+\frac{4m}{wr^3}}+u^2\right)^{\frac{1}{2}}=B_0, \tag{4}$$

where B_0 is an integration constant and $u^r=u<0$ for the inward phantom flow. Further, the energy flux equation can be derived by projecting the energy-momentum conservation law on the four-velocity, that is, $u_\mu T^{\mu\nu}{}_{;\nu}=0$ for which Eq.(3) leads to

$$r^2u\exp\left[\int_{\rho_\infty}^{\rho}\frac{d\rho'}{\rho'+p(\rho')}\right]=-B_1, \tag{5}$$

where $B_1>0$ is another integration constant which is related to the energy flux. Also, ρ and ρ_∞ are densities of the phantom energy at finite and infinite r. From Eqs.(4) and (5), we can obtain

$$(\rho+p)\left(1+wr^2-wr^2\sqrt{1+\frac{4m}{wr^3}}+u^2\right)^{\frac{1}{2}}\exp\left[-\int_{\rho_\infty}^{\rho}\frac{d\rho'}{\rho'+p(\rho')}\right]=B_2, \tag{6}$$

where $B_2=-\frac{B_0}{B_1}=\rho_\infty+p(\rho_\infty)$.

The rate of change of BH mass due to phantom accretion is given by [12]

$$\dot{m} = 4\pi r^2 T^r{}_0. \tag{7}$$

Using Eqs.(5) and (6) in the above equation yields

$$\dot{m} = 4\pi B_1(\rho_\infty + p_\infty). \tag{8}$$

We note that the mass of BH decreases if $(\rho_\infty + p_\infty) < 0$. Thus the accretion of phantom energy onto a BH decreases the mass of BH. It can be noted here that one can solve Eq.(8) for m by using EoS $p = \omega\rho$. Since all p and ρ, violating dominant energy condition, must satisfy this equation, hence it holds in general, that is

$$\dot{m} = 4\pi B_1(\rho + p). \tag{9}$$

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Now, we analyze the critical points (such points at which flow speed is equal to the speed of sound) during the accretion of phantom energy. The phantom energy falls onto BH with increasing velocity along the particle trajectories. The conservation of mass flux is

$$\rho u r^2 = D_0, \tag{10}$$

where D_0 is the constant of integration. Dividing and squaring Eqs.(4) and (10), we obtain

$$\left(\frac{\rho + p}{\rho}\right)^2 \left(1 + wr^2 - wr^2\sqrt{1 + \frac{4m}{wr^3}} + u^2\right) = D_1, \tag{11}$$

where $D_1 = (\frac{B_0}{D_0})^2$ is a positive constant. Differentiating Eqs.(10) and (11) and eliminating $d\rho$, we derive

$$\frac{dr}{r}\left[2V^2 - \frac{wr^2 - wr^2\sqrt{1 + \frac{4m}{wr^3}} + \frac{6m}{r}(1 + \frac{4m}{wr^3})^{\frac{-1}{2}}}{A(r) + u^2}\right] + \frac{du}{u}\left[V^2 - \frac{u^2}{A(r) + u^2}\right] = 0. \tag{12}$$

where $V^2 = \frac{d\ln(\rho+p)}{d\ln\rho} - 1$ and $A(r)$ is the lapse function as defined after Eq.(1).

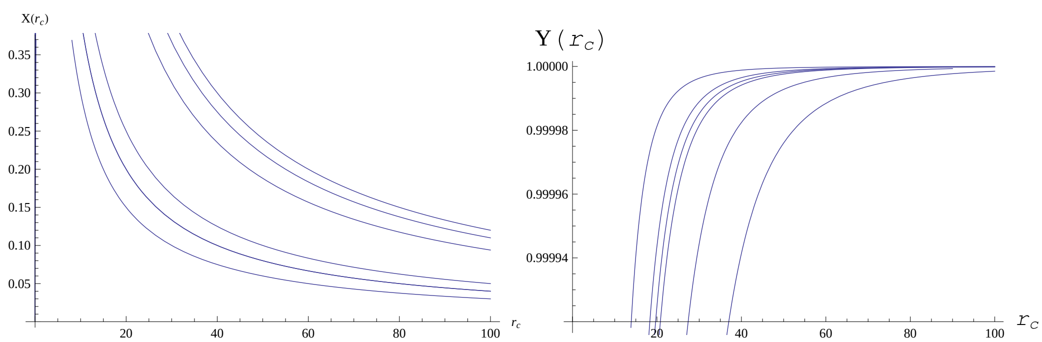

Figure 1: The left right graphs represent the behavior of $X(r_c)$ and $Y(r_c)$, respectively. In both graphs the curves of $X(r_c)$ and $Y(r_c)$ have been plotted for $w = m = 1, 2, 3, 4, 5, 50$ corresponding to up to down order of curves

This equation shows that turn-around points (critical points) are located where both the square brackets vanish. Thus

$$V_c^2 = \frac{wr^2 - wr^2\sqrt{1+\frac{4m}{wr^3}} + \frac{3m}{r}(1+\frac{4m}{wr^3})^{\frac{-1}{2}}}{2(A(r)+{u_c}^2)}, \tag{13}$$

$$V_c^2 = \frac{{u_c}^2}{A(r)+{u_c}^2}. \tag{14}$$

Solving above equations for ${u_c}^2$ and V_c^2, we can obtain

$${u_c}^2 = \frac{1}{2}\left(wr^2 - wr^2\sqrt{1+\frac{4m}{wr^3}} + \frac{3m}{r}(1+\frac{4m}{wr^3})^{\frac{-1}{2}}\right), \tag{15}$$

$$V_c^2 = \frac{\left(wr^2 - wr^2\sqrt{1+\frac{4m}{wr^3}} + \frac{3m}{r}(1+\frac{4m}{wr^3})^{\frac{-1}{2}}\right)}{2+3\left(wr^2 - wr^2\sqrt{1+\frac{4m}{wr^3}} + \frac{m}{r}(1+\frac{4m}{wr^3})^{\frac{-1}{2}}\right)}. \tag{16}$$

We can note that in the limit $w \to \infty$, the above equations reduce to ${u_c}^2 = \frac{m}{2r_c}$ and $V_c^2 = \frac{m}{2r_c - 3m}$, that is the Schwarzschild HB case [9]. The physically acceptable solutions of Eqs.(15) and (16) are obtained if ${u_c}^2 > 0$ and $V_c^2 > 0$, implying that

$$X(r) = \frac{1}{2}\left(wr^2 - wr^2\sqrt{1+\frac{4m}{wr^3}} + \frac{3m}{r}(1+\frac{4m}{wr^3})^{\frac{-1}{2}}\right) > 0, \quad (17)$$

$$Y(r) = 2 + 3\left(wr^2 - wr^2\sqrt{1+\frac{4m}{wr^3}} + \frac{m}{r}(1+\frac{4m}{wr^3})^{\frac{-1}{2}}\right) > 0. \quad (18)$$

It is not possible to determine analytically the values of w, m and r_c, for which $X(r_c) > 0$ and $Y(r_c) > 0$, however we can obtain the positivity of these quantities graphically as shown in Fig. **1**.

3 Summary

It is known that a BH surrounding the matter is expected to capture particles of a matter that passes near BH. In this paper, we have explored the phantom accretion onto a BH in HL gravity. By assuming that infalling fluid does not alter the generic properties of the BH, the equations of motion for steady state spherically symmetric phantom flow near BH have been derived. We herein discuss the accretion and critical accretion onto a BH by using Babichev-Dokuchaev-Eroshenko method [10]. It has been found that like the cases of the Schwarzschild and RN BHs, phantom accretion decreases the mass of BH in HL gravity. There exist two horizons for the HL BH, which depends on

Table 1: Location of horizons for the values of m and w

$m = w$	r_-	r_+
1	0.2928	1.7072
2	0.1291	3.8708
3	0.0845	5.9145
4	0.0629	7.9370
5	0.0146	9.9853
6	0.0839	11.9581
7	0.0358	13.9642
8	0.0313	15.9686
50	0.0018	99.9949

m and w. The critical accretion analysis implies that accreting fluid would

attain speed of sound if $1 \leq m = w \leq 50$. The quantities $X(r)$ and $Y(r)$ plotted in Fig. **1**, imply that for $1 \leq m = w \leq 50$, the critical accretion is possible as $V_c^2 \geqslant 0$. The location of critical point in case is $15 \leqslant r_c \leqslant 100$. Thus the location of inner and outer horizons (Eq.(2)) for $1 \leq m = w \leq 50$ is given by Table **1**.

From the Fig. **1** and Table **1** , we observe that for $1 \leq m = w \leqslant 7$, the critical accretion points lie outside the inner and outer horizons. For $7 < m = w < 50$, the critical accretion pints lies inside the outer horizons but outside inner horizons. Thus, we conclude that for each value of $1 \leq m = w \leqslant 7$, there exist two circular horizons (inner and outer) that are bounded by a circle of larger radius, representing the critical accretion region. As m becomes large $\leqslant 50$ the outer horizon lies into the critical accretion region.

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